Bounded rationality and unemployment dynamics

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Abstract

Using the bounded rationality implementation developed in Evans, Evans, and McGough (2021), we consider unemployment dynamics driven by aggregate productivity shocks within a McCall-type labor-search model. We find that bounded rationality magnifies the impact effect of a decline in productivity on unemployment. Over the course of a recession, bounded rationality induces excess pessimism, resulting in higher unemployment relative to the rational model.

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1 Introduction

Search theoretic models of the labor market provide an important approach to explaining unemployment dynamics, for example see Rogerson, Shimer, and Wright (2005). Despite their attractiveness, these models struggle to match several important moments in the data when agents are taken as rational. A particularly notable example is the "Shimer Puzzle," which states that model-generated fluctuations in the unemployment rate over the business cycle are much smaller than found in the data: see Shimer (2005).

In Evans, Evans, and McGough (2021) we introduced bounded rationality into the McCall model. After establishing theoretical results showing asymptotic convergence to fully rational behavior, we investigated the implications of boundedly optimal decision making on unemployment dynamics. In particular, we established that our modeling approach has the potential to explain the frictional wage dispersion puzzle exposed by Hornstein, Krusell, and Violante (2011). In the current paper we examine whether the application of this approach, to a version of the model with aggregate productivity shocks, moves the needle on the Shimer puzzle.

To capture the business cycle within the McCall framework, we assume agents' wages are in part driven by a 2-state Markov process calibrated to match business cycle frequencies. The reservation wages of rational agents depend on the aggregate state and fully reflects the state-dependent distributions of wage offers. Our boundedly rational agents do not know distributions of wage offers and do not observe the aggregate state. Instead they adapt their reservation wages based on recent experience.

Relative to the rational counterpart, our model's unemployment dynamics exhibit two striking features. Boundedly rational agents are slower to adjust their beliefs to aggregate movements in productivity; therefore, at the onset of a bust they are, in effect, overly optimistic about the distribution of wage draws. Consequently they keep their reservation wages high, which magnifies the impact effect on unemployment. If the bust continues for an extended period, boundedly rational agents become overly pessimistic: in effect, they come to believe that the wage distribution has permanently and significantly deteriorated. As a result, the difference between unemployment rates in sustained booms and sustained busts is magnified under bounded rationality.

2 The model

A version of the McCall (1970) model was used by Evans, Evans, and McGough (2021) to study bounded optimality in a labor search environment. Here, we extend our version of the model to include time-varying productivity shocks. An infinitely-lived agent receives utility from consumption via the instantaneous utility function U. Time is discrete, wages are paid in perishable goods, and there is no storage technology. The *wage offer* W is the product of the idiosyncratic component $\hat{w}$, referred to as the *match productivity*, and a time-varying aggregate productivity shock z; thus $W = \hat{w} \cdot z$. We take z as following a 2-state Markov process with states $z_L < 1 < z_H$ and transition matrix P.

We first discuss rational behavior. At the beginning of a given period the rational agent observes the aggregate state z and decides whether or not to accept the wage offer $W = \hat{w} \cdot z$. The match productivity $\hat{w}$ is drawn from a distribution that depends on whether the agent was employed or unemployed at the end of the previous period. If the agent was employed, her match productivity in the previous period constitutes her match productivity in the current period. If the agent was unemployed at the end of the previous period, she receives a match productivity $\hat{w}$ drawn from a time-invariant exogenous distribution F (density dF). In either case, the agent must decide whether or not to accept the wage offer W.

If the wage offer is not accepted the agent is unemployed in the current period, and receives an unemployment benefit $b > 0$. If the offer is accepted then the agent receives the wage W in the current period. We assume exogenous job destruction parameterized by $\alpha \in (0,1)$. At the end of the period, with probability $1-\alpha$ the match with the firm is preserved and, because she is employed at the end of the current period, her match productivity remains unchanged the following period. With probability α the match is destroyed and at the beginning of the next period the agent's new match productivity is drawn from F.

We assume that U is increasing and concave, F has support $[w_{\min}, w_{\max}]$, and all match productivity draws $\hat{w}$ are independent over time and across agents. Let $V^*(W,z)$ be the rational agent's value of receiving wage offer W when the state is z, and define $Q^*(z) = E\,V^*(\hat{w} \cdot z, z)$, where the expectation is taken over match productivities. We note that $Q^*(z)$ measures the value assigned by the agent, in aggregate state z, of being in the unemployed state and thus facing a random wage draw at the beginning of the period. The rational agent's program may be written as follows:

$$V^*(W,z) = \max\left\{U(b) + \beta E\left(Q^*(z')|z\right), U(W) + \alpha\beta E\left(Q^*(z')|z\right) + (1-\alpha)\beta E\left(V^*\left(z'W/z, z'\right)|z\right)\right\}.$$

It is well known (e.g. see Lemma 1 in the Appendix) that the rational agent's behavior is determined by a *reservation wage* W^* that depends on the aggregate state z: the agent accepts her wage offer W exactly when it exceeds W^*.

Remark 1. For our calibration, the reservation wages satisfy

$$\frac{z_L}{z_H} W^*(z_H) < W^*(z_L) < W^*(z_H).$$

Intuitively, the wage W offered when $z = z_L$ implies a wage path that strictly dominates the path implied by the same wage W offered when $z = z_H$, so that $W^*(z_L) < W^*(z_H)$. To see the first inequality, let $\hat{w}_H^* = W^*(z_H)/z_H$ be the *reservation match productivity* in the boom, and similarly for $\hat{w}_L^*$. A given match productivity $\hat{w}$ has higher contemporaneous return relative to unemployment benefits in a boom. This leads to the agent being more selective during a bust, i.e. $\hat{w}_H^* < \hat{w}_L^*$, which is equivalent to the first inequality. See the Appendix for a more detailed development of this remark.

Now we turn to boundedly rational behavior which departs from full rationality in two distinct ways. First, the boundedly rational agent is either unaware of, or anyway fails to explicitly account for the impact of aggregate productivity on her future wages. And second, we adopt the bounded optimality approach to decision making emphasized by Evans and McGough (2018) and Evans, Evans, and McGough (2021), in which agents make decisions based on *perceived* trade-offs.

To implement our approach, denote by Q the agent's current perceived (i.e. subjective) value of receiving a random wage offer, and let $V(W,Q)$ denote the perceived value of holding wage offer W. Boundedly optimal agents make decisions by solving the following optimization problem

$$V(W,Q) = \max\left\{U(b) + \beta Q, U(W) + \beta(1-\alpha)V(W,Q) + \beta\alpha Q\right\}.$$

Note that if an agent accepts the wage offer W then $V(W,Q) = \phi U(W) + \beta\alpha\phi Q$, where $\phi = (1-\beta(1-\alpha))^{-1}$. The agent's *reservation wage* $\bar{W}(Q)$ is defined implicitly via

$$U(b) + \beta Q = \phi U\left(\bar{W}(Q)\right) + \beta\alpha\phi Q. \tag{1}$$

A boundedly rational agent with beliefs Q accepts wage offer W exactly when $W > \bar{W}(Q)$.

It remains to detail how agents update beliefs as new data become available. We adopt the adaptive learning approach introduced by Bray and Savin (1986) and Marcet and Sargent (1989), and employed in a wide range of applications in macroeconomics and finance including, for example, Kasa (2004), Eusepi and Preston (2011), Slobodyan and Wouters (2012), Adam, Marcet, and Nicolini (2016), Branch, Petrosky-Nadeau, and Rocheteau (2016), Williams (2018), and Honkapohja and Mitra (2020).

For simplicity we assume that both unemployed and employed agents observe one random wage offer each period. Let Q_t be the value, perceived at the start of period t, of being unemployed. Noting that Q_t measures the agent's perception of the value of receiving a random wage draw, we assume that an agent who observes wage offer W_t updates her beliefs Q_t at the end of period t according to the algorithm

$$Q_{t+1} = Q_t + \gamma_{t+1}\left(V(W_t, Q_t) - Q_t\right). \tag{2}$$

Here $\gamma_t \in (0,1)$ is the *gain* sequence, which measures the weight placed on new information. If the gain decreases to zero at an appropriate rate it is possible to show that beliefs converge almost surely to the *restricted perceptions value* $\bar{Q}$, which can be viewed as the optimal time-invariant beliefs.[1] We focus on the constant gain case, $\gamma_t = \gamma$, which is known to be useful for tracking structural change, here taking the form of switches between productivity regimes.

[1]The restricted perceptions value $\bar{Q}$ is the partial equilibrium analog to a *restricted perceptions equilibrium*: see Branch (2006) for a nice survey.

3 Unemployment dynamics

We now take our model as populated by many agents, which allows for analysis of interesting aggregates including the unemployment rate. In the rational model, the unemployment rate dynamics are given by

$$u_t = (1-h_t)\,u_{t-1} + (1-u_{t-1})\,(\alpha + (1-\alpha)q_t)\,,$$

where h_t is the *hazard rate* of leaving unemployment, i.e. the probability per period of an unemployed agent becoming employed, and q_t is the *quit rate*, which measures the proportion of agents employed in period $t-1$ who reject their wage offers in period t.[2] We note that the hazard rate depends on z_t and the quit rate exhibits history dependence based the distribution of accepted wage offers.

In the boundedly rational model the unemployment rate in a given period depends on the joint distribution of beliefs and employment status across agents as well as on the aggregate state. We use simulations to study the implied dynamics.

We use the calibration from Evans, Evans, and McGough (2021). The time unit is months and the discount rate is $\beta = 0.996$, the monthly separation rate is $\alpha = 3\%$, and $b = \$31,200$ giving a replacement rate of 41%. Finally, utility is *CRRA* with parameter $\sigma = 3.25$ to match a job-finding rate of 43%. The exogenous wage distribution is assumed lognormal with shape parameters $\mu = 11.0$, $s = 0.25$, which yields a median household wage of approximately $\$60,000$.[3]

Following Krusell and Smith (1998), the productivity shocks are $\pm 1.0\%$, and the transition matrix is tuned to accord with median cyclic durations in the post-war era: a median boom length of 58 months and a median bust length of 10 months.[4] Gray regions in the figures correspond to (simulated) recessions, and simulations were initialized in a bust.

For boundedly rational simulations we again follow Evans, Evans, and McGough (2021) and take an economy populated with 100,000 agents. We set the gain $\gamma = 0.1$.[5] Agents' beliefs are initialized by simulating the economy in a bust state for an extended period.

Figure 1 provides evidence for the amplification of business cycles induced by bounded rationality. While the unconditional mean unemployment rates for the rational and boundedly rational economies are the same, at 6.67%, the simulated productivity shocks induce much more volatility in the bounded rationality (BR) economy. Two features of the BR model's

[2] Within the context of our model, it is natural to refer to these endogenous separations as quits. In more general models, with bargaining over surpluses, these endogenous separations would be mutual.

[3] The value of s gives an interquartile income range of $\$50,583$ to $\$70,871$.

[4] November 1948 to February 2020. Source: https://www.nber.org/cycles/cyclesmain.html

[5] Evans, Evans, and McGough (2021) discuss gain values at length and focus on gains of 0.05 and 0.1. We choose the higher value here because of the agents' need to account for structural change. Simulations with $\gamma = 0.05$ yield very similar results.

unemployment dynamics distinguish it from its rational expectations (RE) counterpart: an amplified impact response in case of negative shocks, and a more persistent *medium run* phenomenon discussed below.

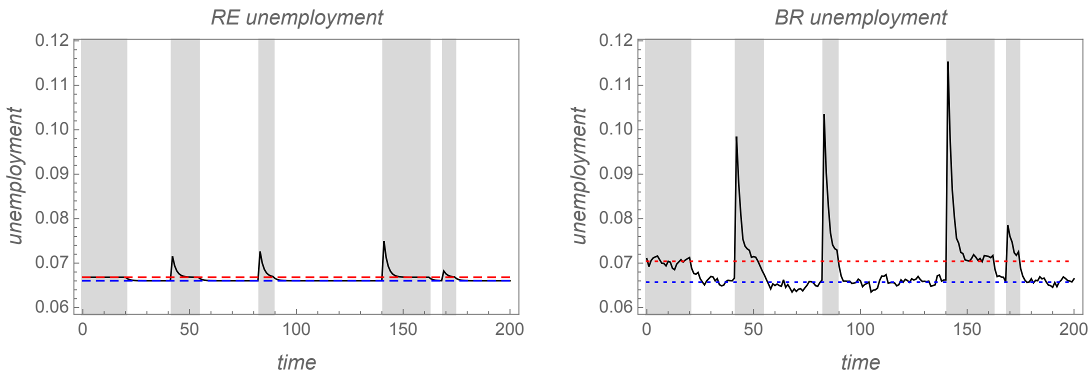

Figure 1: Unemployment dynamics with aggregate productivity shocks. Shaded strips indicate recessions and horizontal lines identify medium-run unemployment levels. The left panel demonstrates the Shimer puzzle, the muted unemployment response of the RE model to productivity shocks, and the right panel shows the potential for BR to resolve the puzzle.

To understand the amplified response on impact, first consider the behavior of rational agents entering a recession after an extended boom. As noted in Remark 1, the fall in wages caused by the decline in aggregate productivity is larger than the fall in the reservation wage. Therefore, some agents who were employed during the boom will quit at the onset of the bust because their reduced wages now fall below their new reservation wage. This causes the spikes in unemployment seen in the left panel of Figure 1.

Just as with rational agents, the wages of employed boundedly rational agents fall at the onset of a bust; however, BR agents do not adjust their beliefs (and thus reservation wages) on impact. Therefore more BR agents find that their reduced wages fall below their reservation wage, which leads to the much larger unemployment spikes seen in the right panel of Figure 1. In effect, BR agents are initially overly optimistic about their future wages draws and so are more willing to enter into unemployment rather than accept their lower wages.

The *medium-run* unemployment levels associated with booms and busts are defined as the levels obtained after the economy has been in a boom or a bust for an extended period. These levels are indicated by the horizontal (dashed) lines, with the upper (red) lines corresponding to busts. The inequalities given in Remark 1 imply that medium run unemployment rate is higher in a bust than in a boom when are agents are rational. The same relationship holds in the BR case, and is, in fact, amplified: as a bust (boom) persists, BR agents become overly pessimistic (optimistic) about the distribution of wage draws.

4 Conclusion

While our results have been obtained within the partial equilibrium framework of the McCall model, the mechanisms at play strongly suggest that analogous results will arise in general equilibrium environments, e.g. Diamond-Mortensen-Pissarides models. This is the subject of current research.

Appendix

The inequalities given in Remark 1 are satisfied in our calibrated model. While a proof for generic calibrations is not available, Proposition 1 below establishes the result for a special case. Specifically, Proposition 1 shows that, in the rational model and for transition matrices close enough to i.i.d., the onset of a bust leads to a fall in the reservation wage that is proportionally smaller than the corresponding fall in the wages of employed agents. It follows that this fall in reservation wages leads to a spike in unemployment.

It is helpful to redefine the rational agent's state as $(\hat{w},z)$ instead of (W,z). The value, $v^*(\hat{w},z)$, of having a match productivity $\hat{w}$ if the aggregate state is z then solves the following Bellman equation

$$v^*(\hat{w},z) = \max\left\{U(b)+\beta\mathbb{E}\left[Q^*(z')|z\right], U(\hat{w}z)+\beta\mathbb{E}\left[\alpha Q^*(z')+(1-\alpha)v^*(\hat{w},z')|z\right]\right\} \quad (3)$$

$$Q^*(z) = \int v^*(\hat{w},z)dF(\hat{w}). \quad (4)$$

The following Lemma confirms the standard properties of the worker's value function and decision rules.

Lemma 1 *The value function $v^*(\hat{w},z)$ that solves (3)-(4) is continuous and weakly increasing in $\hat{w}$. There exists a reservation productivity $w^*(z)$ such that the worker accepts all wage offers with productivity greater than $w^*(z)$. For all $\hat{w} > w^*(z)$, $v^*(\hat{w},z)$ is strictly increasing in $\hat{w}$.*

Proof. Let $v \to T(v)$ be the Bellman map associated with the maximization problem (3)-(4). Suppose that $v(\hat{w},z)$ is weakly increasing and continuous in $\hat{w}$. As

$$U(\hat{w}z)+\beta\mathbb{E}\left[\alpha Q^*(z')+(1-\alpha)v(\hat{w},z')|z\right] \quad (5)$$

is then strictly increasing and continuous in $\hat{w}$ and $U(b)+\beta\mathbb{E}[Q^*(z')|z]$ is constant we can conclude that $T(v)$ is weakly increasing and continuous in $\hat{w}$. Standard approaches imply that T is a contraction and since the set of weakly increasing and continuous functions is closed we have that the unique fixed point of T is weakly increasing and continuous in $\hat{w}$.

The remaining two claims follow directly from our result that

$$U(\hat{w}z)+\beta\mathbb{E}\left[\alpha Q^*(z')+(1-\alpha)v^*(\hat{w},z')|z\right]$$

is strictly increasing and continuous in $\hat{w}$. ■

With the properties of Lemma 1 in hand we are able to show that for transition matrices close enough to i.i.d. the onset of a bust leads to a fall in the reservation wage that is smaller than the corresponding fall in productivity.

Proposition 1 *Let $\mathscr{P}$ be the set of all 2×2 i.i.d. transition matrices and $d_{\mathscr{P}}(P)$ be the minimum distance from P to an element of $\mathscr{P}$. There exists $\gamma > 0$ such that $d_{\mathscr{P}}(P) < \gamma$ implies $z_L W_H^*/z_H < W_L^* < W_H^*$.*

Proof. We will show that $z_L W_H^*/z_H < W_L^* < W_H^*$ holds for all i.i.d. transition matrices. The result then follows by continuity. When P is i.i.d., the reservation productivity $w^*(z)$ must satisfy

$$U(w^*(z)z) + \beta\mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(w^*(z), z')\right] = U(b) + \beta\mathbb{E}\left[Q^*(z')\right].$$

Define

$$g(\hat{w}, z) = U(\hat{w}z) + \beta\mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(\hat{w}, z')\right].$$

Since v^* is weakly increasing in $\hat{w}$ and $z_L < z_H$, we have that $g(\hat{w}, z_L) < g(\hat{w}, z_H)$ for all $\hat{w} > 0$, and that $g(\hat{w}, z)$ is strictly increasing in $\hat{w}$. As $U(b) + \beta\mathbb{E}[Q^*(z')]$ is independent of z, we conclude that $w^*(z_L) > w^*(z_H)$, which implies the first inequality: $z_L W_H^*/z_H < W_L^*$.

As $z_L w^*(z_L) = W_L^*$ and $z_H w^*(z_H) = W_H^*$ we conclude that

$$U(W_L^*) + \beta\mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(w^*(z_L), z')\right] = U(W_H^*) + \beta\mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(w^*(z_H), z')\right].$$

Lemma 1 implies that $v(\hat{w}, z)$ is weakly increasing in $\hat{w}$ and $v^*(\hat{w}, z_H)$ is strictly increasing for all $\hat{w} > w^*(z_H)$. As $w^*(z_L) > w^*(z_H)$, we can immediately conclude that

$$\mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(w^*(z_L), z')\right] > \mathbb{E}\left[\alpha Q^*(z') + (1-\alpha)v^*(w^*(z_H), z')\right],$$

and hence $W_L^* < W_H^*$ as desired. ■

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